# Short Takes 331

## Roots using the fixed-point method

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With Newton-Raphson,

$$x_{new} = x_{old} - \frac{f(x_{old})}{f'(x_{old})}$$

Then, iterate until $x_{new} \approx x_{old}$ to some required precision.

The idea of the fixed-point method is very similar...

$$f(x) = 0 \longrightarrow x = g(x) \quad \text{for some } g(x)$$

There will be many ways to do this!

**E.g.**

$$\underbrace{x^2 + x - \cos x = 0}_{f(x)} \implies x = \underbrace{\cos x - x^2}_{g(x)}$$

or

$$x = \underbrace{\pm\sqrt{\cos x - x}}_{g(x)}$$

**Key point:**

We must have

$$|g'(x)| < 1 \longrightarrow \text{This defines a region of } x \text{ values}$$

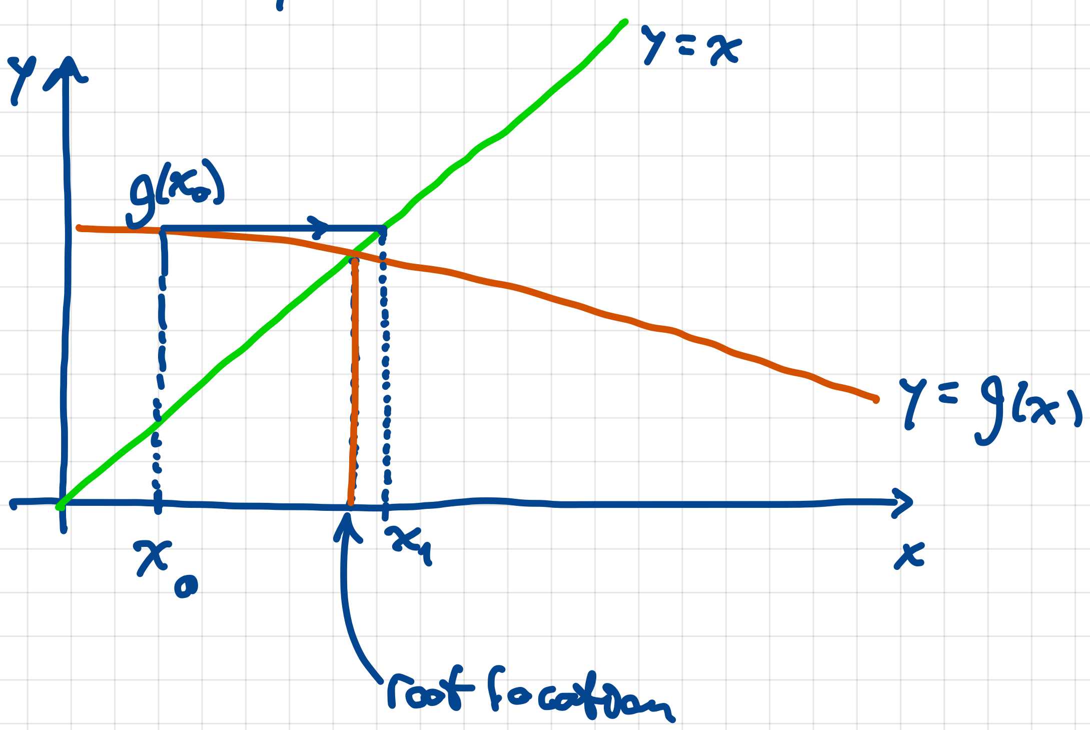
for the method to converge.

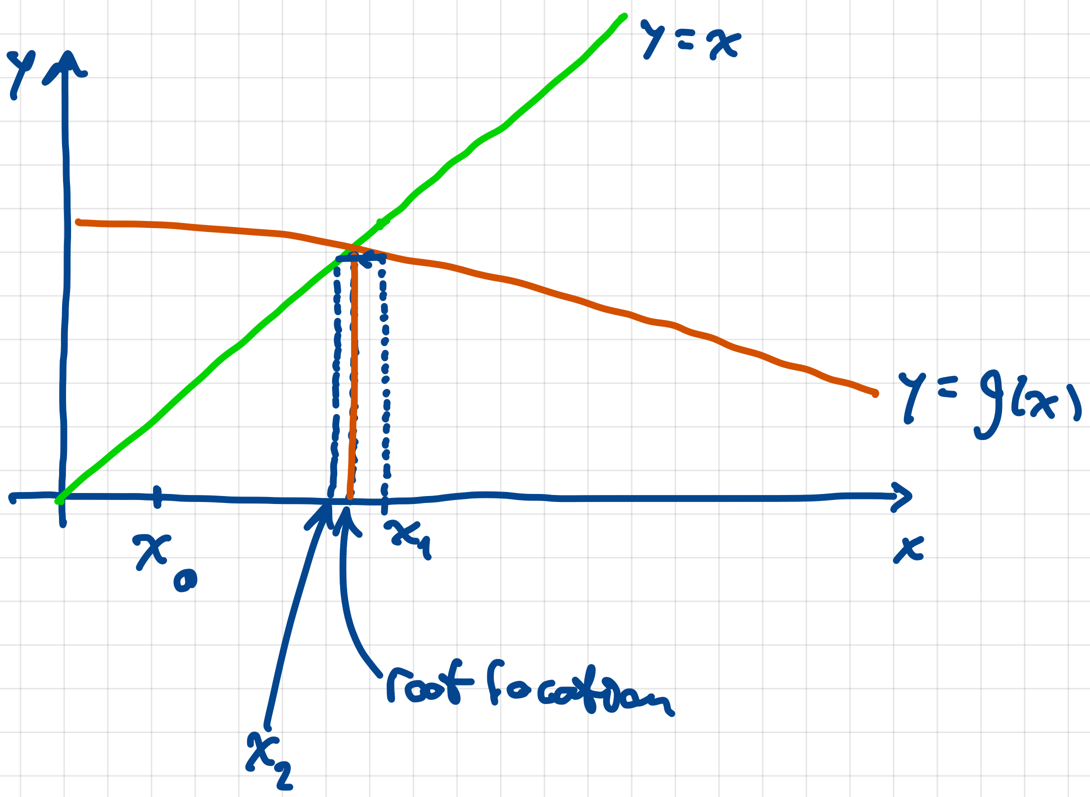
- We need our root to be in that region, as well as our starting point $x_0$.

- How to proceed? Very simple!
  - Pick initial $x_0$
  - Calculate $g(x_0)$
  - Set $x_1 = g(x_0)$ and return to the first step using $x_1$ as $x_0$.

As this process converges, we approach $x = g(x)$.

Geometrically...




- When drawing this process:
  - from x axis to g(x)
  - from g(x) to y = x
  - back to x axis.

- Note Newton-Raphson is a type of fixed-point method where

$$g(x) = x - \frac{f(x)}{f'(x)}$$

- Our convergence criterion becomes

$$\left| \frac{f(x)\, f''(x)}{\left(f'(x)\right)^2} \right| < 1$$

- Derive it!
- How does it change when root approach is not linear?