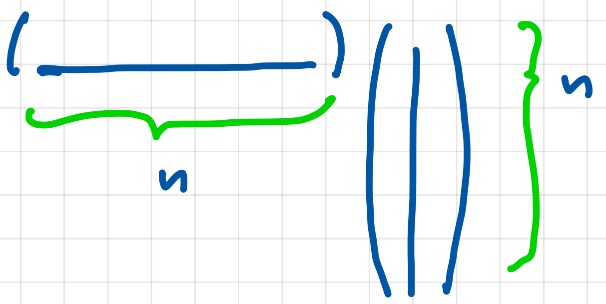
Short Takes 331

Matrix multiplication (part 2): mechanics & cost

Suppose $A \in \mathbb{R}^{m \times n}$, $\vec{v} \in \mathbb{R}^n$, $\vec{w} \in \mathbb{R}^n$

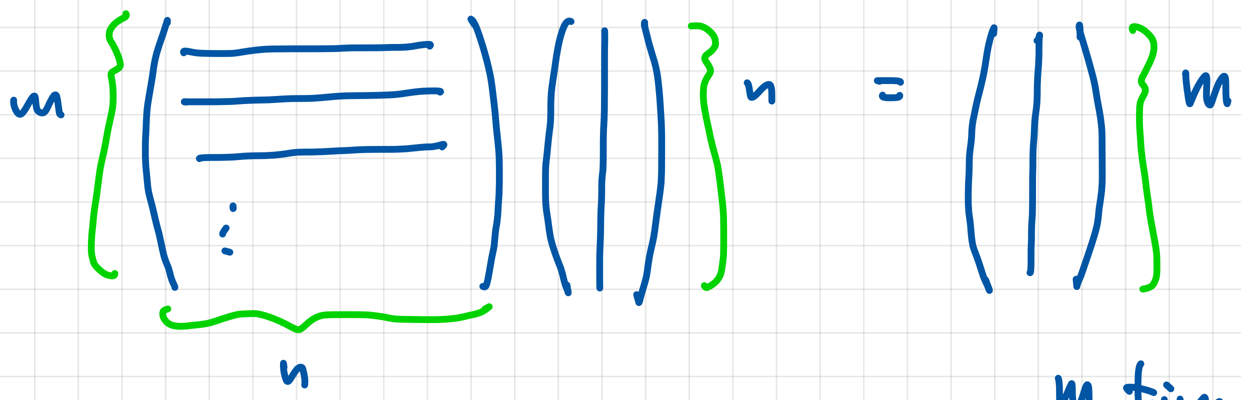
- $\vec{v} \cdot \vec{w} = c \qquad c \in \mathbb{R}$



$\rightarrow$ <u>n multiplications & additions</u>

$O(n)$

- $A\vec{v} = \vec{u} \quad , \quad \vec{u} \in \mathbb{R}^m$



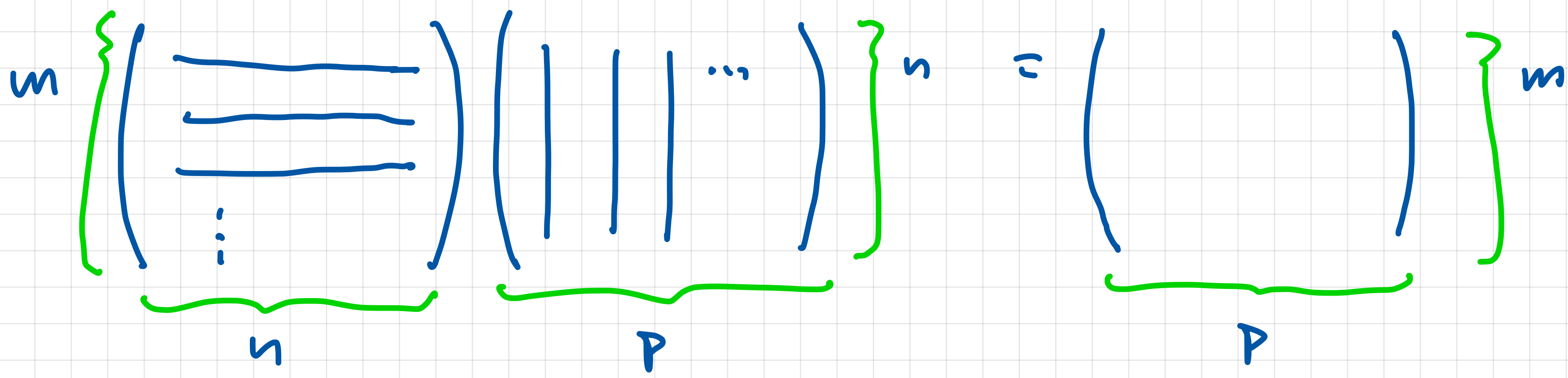
$\rightarrow$ <u>m times</u>

<u>n multiplications & additions</u>

$O(m \times n)$

If A is square of size $n \times n$ $\rightarrow O(n^2)$

- Similarly, matrix-matrix products require $O(m \times n \times p)$ if the matrices involved are of size $m \times n$ and $n \times p$. For square matrices this becomes $O(n^3)$



• Can we do better?

Take for example matrix-vector multiplication ...

$\underbrace{n \text{ times}}_{?}$ $\underbrace{\text{vector-vector } n\text{-component multiplication}}_{?}$

We can improve on this in some special cases
e.g. Fourier transform
$O(n\log n)$

We can improve on this e.g. if there are many zeros: sparse matrices, such as

$$\begin{pmatrix} 1 & -1 & 0 & \cdots & & \\ 0 & 1 & -1 & 0 & \cdots & \\ 0 & 0 & 1 & -1 & 0 & \cdots \\ & & & \ddots & \ddots & \ddots \end{pmatrix}$$

Here the cost of vector-vector mult. is 2, not n.

$\rightarrow O(2n)$ instead of $O(n^2)$

→ It's important to know different kinds of matrices and their properties. ⟶ next video!

→ This will help us solve linear problems in physics.

$$A\vec{x} = \vec{b}$$

rhs (usually external excitation)

unknown (physics we want)

how your system responds (EM, fluids, etc + boundary conditions)

- <u>Examples</u>

$$\nabla^2 \phi = -f \qquad \text{Poisson eq.}$$

$$\Box \psi = f \qquad \text{Wave eq. (w/ source)}$$

$$\left(m\frac{d^2}{dt^2} + k\right)x = F(t) \qquad \text{Forced HO}$$

In all these cases our job is to somehow put the diff. op. on the other side of the eq. to find the unknown. That amounts to finding the inverse.
