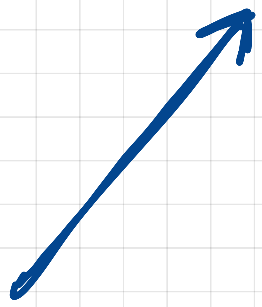
Short Takes 331

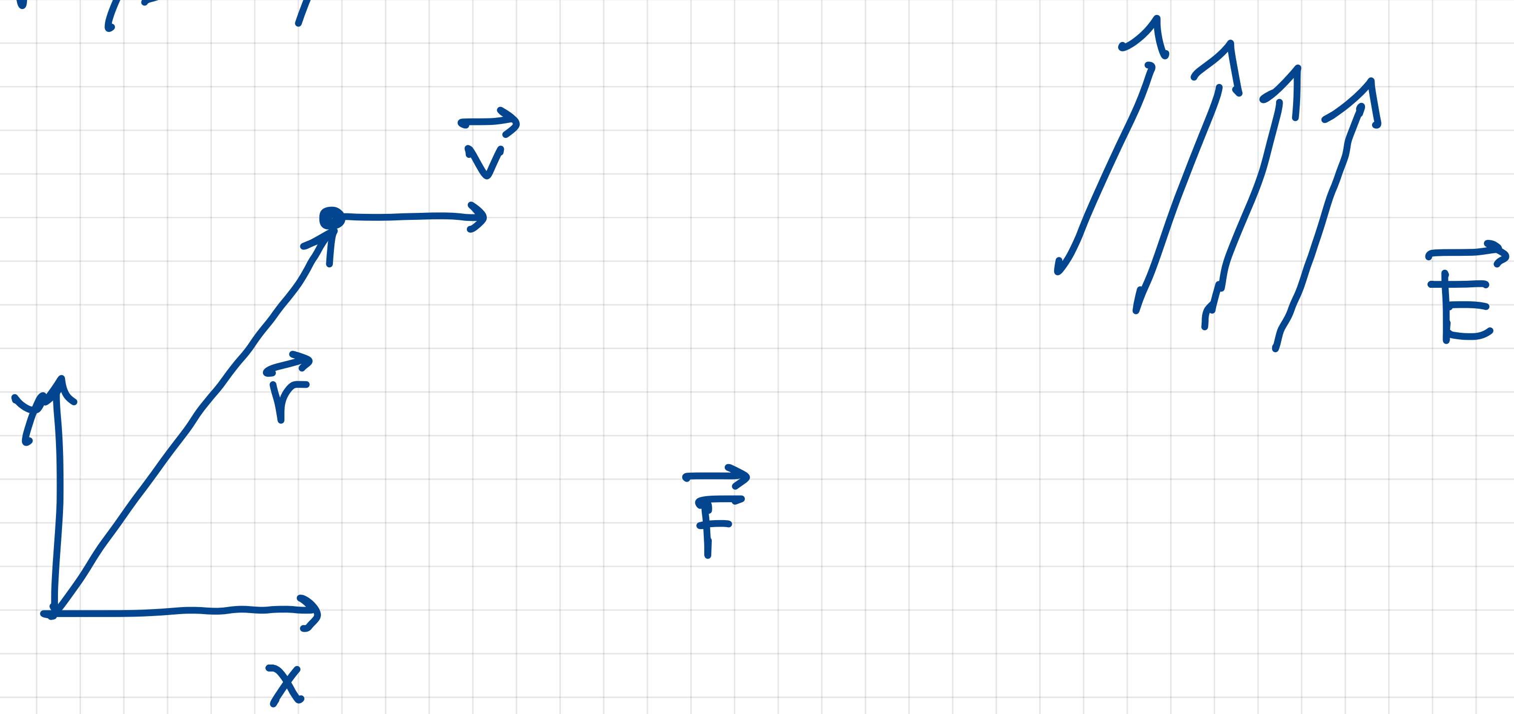
→ What is a vector?

→ Graphically ...



- magnitude
- direction

→ Physically ...



→ Mathematically ...

An element of a <u>vector space</u> V

→ What is a vector space?

A set of <u>objects</u> that respect the following rules...

???

- Think of it as trying to define the bishop piece in the game of chess.

  You need to explain the setting (game) at least briefly and then define the bishop by its properties on the board (it can only move diagonally).

- Similarly, vectors are defined in terms of their properties in the "setting" of a vector space.

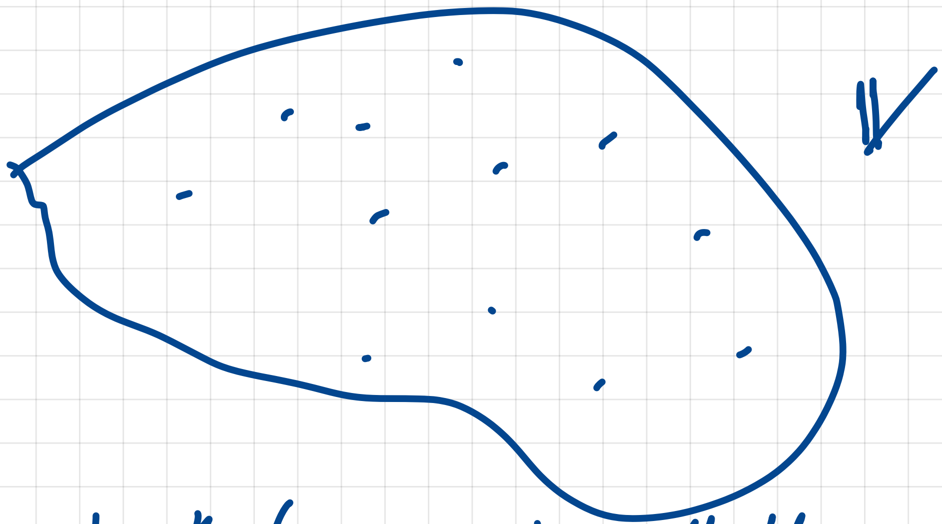
  So...

  → What is a vector space?

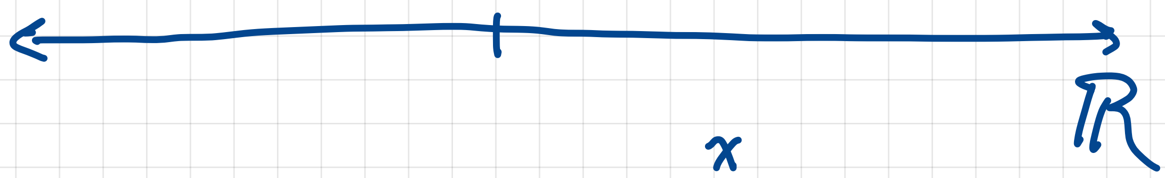
  - We need <u>two things</u>:

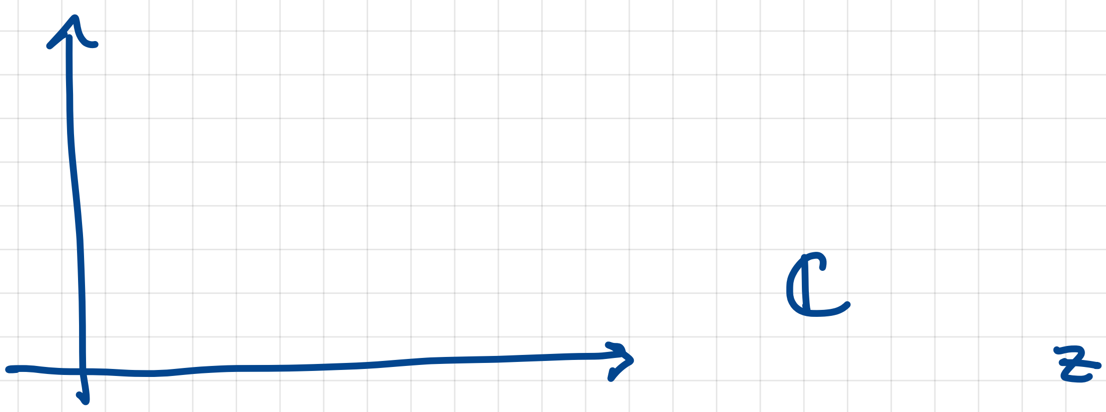
    → A set of "objects" (we will call them <u>vectors</u>)

    $\vec{v}, \vec{w}, \vec{r}, \ldots$



    → A set of "numbers" (we will call them <u>scalars</u>)





generically $\mathbb{S}$

→ These scalars and vectors must obey certain rules.

→ Scalars must form a "field"

→ must have well-defined addition "+" operation

$x, y \in S$

$x+y$ also $\in S$

→ must have well-defined multiplication "x" operation

$x, y \in S$

$x \times y$ also $\in S$

→ Both operations must have inverses and neutral elements in S.

This means there is a "zero" and a "one" in S

$x + 0 = x \quad , \quad 0 \in S$

$x \times 1 = x \quad , \quad 1 \in S$

$x^{-1} \in S \quad , \quad -x \in S$

→ Other properties

- Associativity $\quad a+(b+c) = (a+b)+c$
- Commutativity $\quad ab = ba$, $\quad a+b = b+a$
- Distributivity $\quad (a+b)c = ac + bc$

→ Vectors must obey:

- Addition is "closed"

$$\vec{v} + \vec{w} \in \mathbb{V} \quad \text{if} \quad \vec{v}, \vec{w} \in \mathbb{V}$$

- Neutral element $\vec{0}$ is in $\mathbb{V}$ as well.

- Multiplication by a scalar is also closed:

  $a \in \mathbb{S}$

  $\vec{v} \in \mathbb{V}$, then $a.\vec{v} \in \mathbb{V}$.

- Other properties:
  - vector addition is commutative and associative, and obeys the distributive property when multiplying by a scalar.

Examples

$\mathbb{R}^2$: (x, y) , $x, y \in \mathbb{R}$ — $(1, 2)$ $(7, 5.4)$ ...

with $\mathbb{R}$ as scalars — $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\mathbb{C}^2$: (z, w) , $z, w \in \mathbb{C}$ — $\begin{pmatrix} 1+i \\ \pi \end{pmatrix}$ etc

with $\mathbb{C}$ or $\mathbb{R}$ as scalars

$\mathbb{R}^n$, $\mathbb{C}^n$

Note: there is no notion of vector-vector multiplication in our definition! Neither $\vec{v} \times \vec{w}$ nor $\vec{v} \cdot \vec{w}$.