Short Takes 331

Fourier series: example 2

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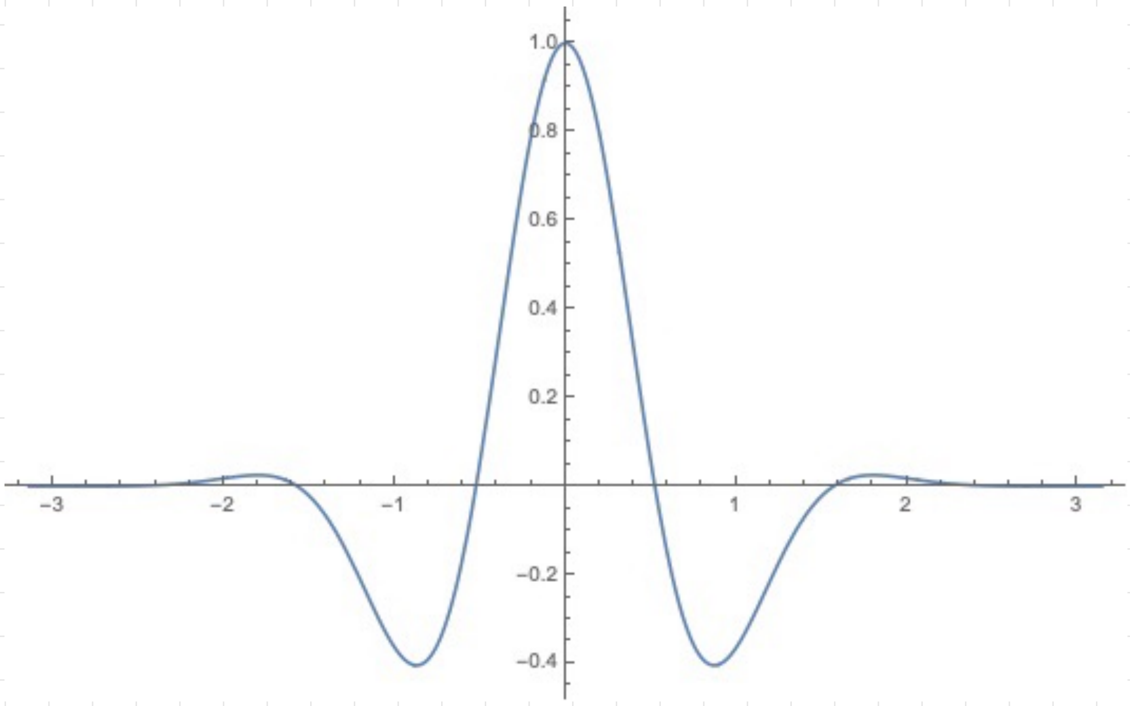
$$f(x) = e^{-x^2}\cos(3x) \quad , \quad x \in [-\pi, \pi]$$



$$f(x) = \frac{A_0}{2} + \sum_{k=1}^{\infty}\left(A_k \cos\left(\frac{k\pi x}{L}\right) + B_k \sin\left(\frac{k\pi x}{L}\right)\right)$$

$$= \frac{A_0}{2} + \sum_{k=1}^{\infty}\left(\underline{A_k \cos(kx)} + \cancel{B_k \sin(kx)}\right)$$

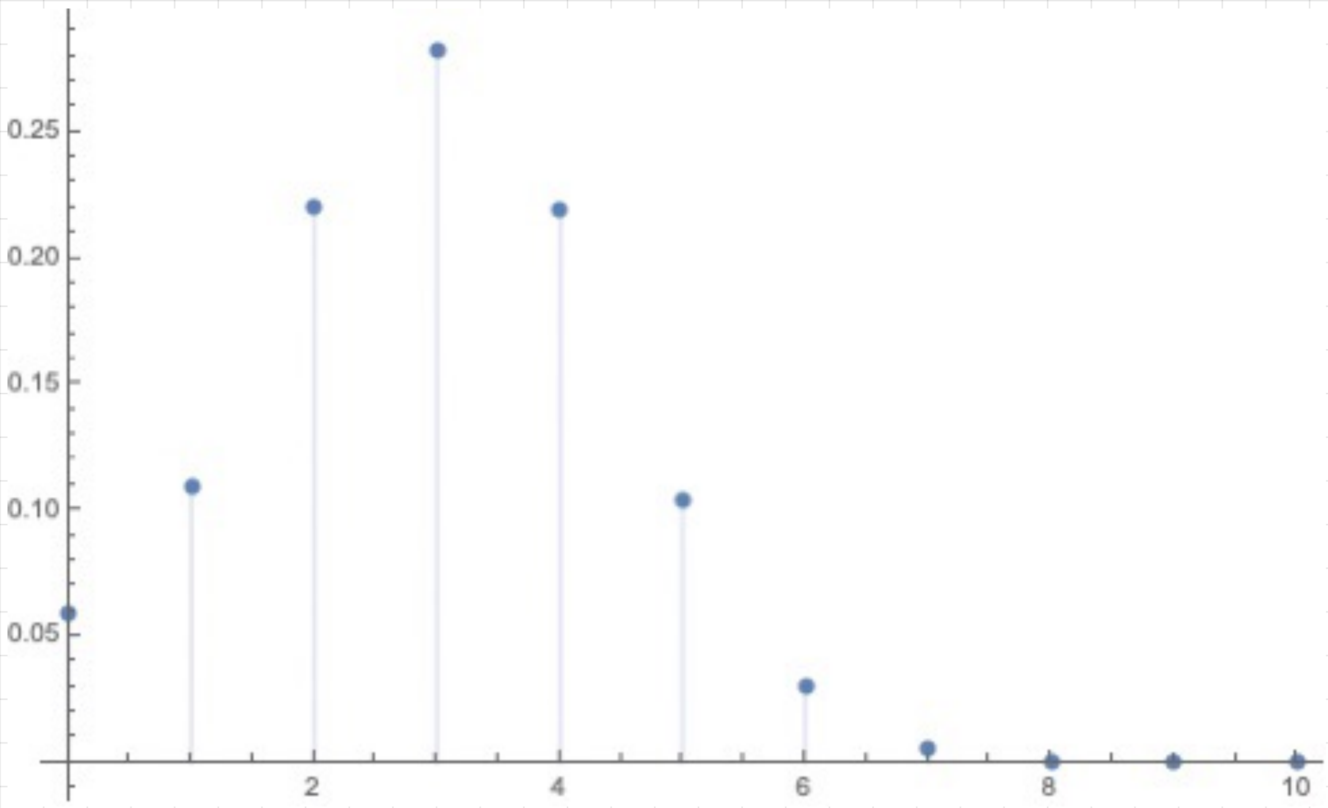
where
$$A_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$A_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx$$

$$B_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,dx \longrightarrow 0$$

- What can we expect?

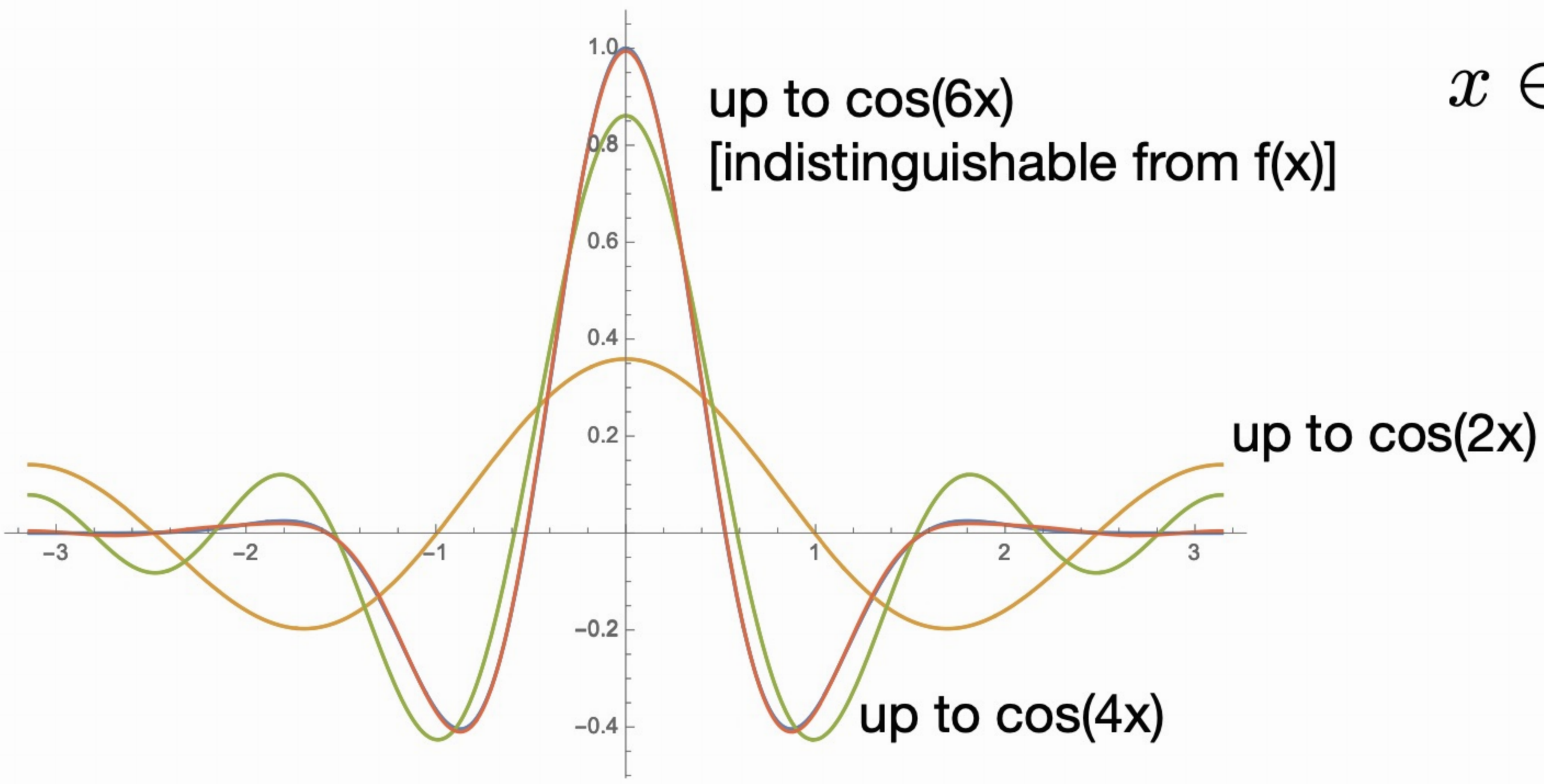
$$\underline{A_0},\ \underline{A_1},\ \underline{A_2},\ \underline{A_3},\ \underline{A_4},\ \underline{A_5},\ \underline{\ldots}$$



k_0 k_1 k_2 $\dots$

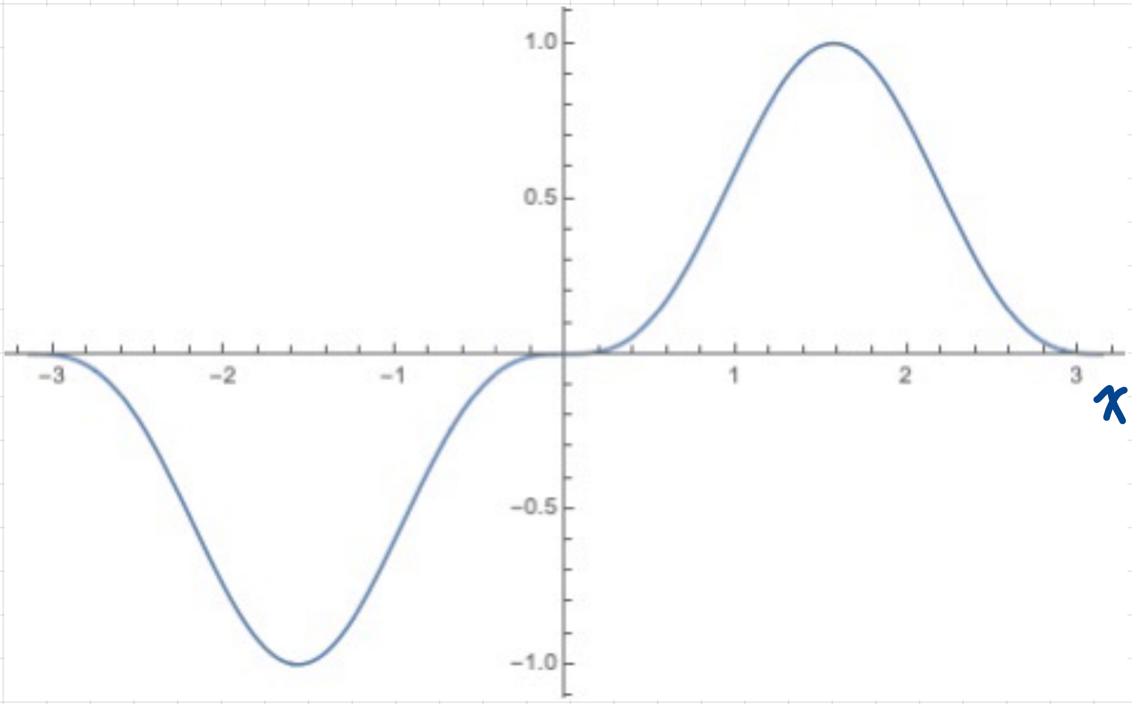
$$f(x) = e^{-x^2}\cos(3x)$$
$$= 0.02973 + 0.10893\cos(x) + 0.22024\cos(2x) + \dots$$

$x \in [-\pi, \pi]$



The actual example ...

$\sin^3(x)$, $x \in [-\pi, \pi]$



More than one way to do this, even without integrals!

Our goal:

$$\sin^3(x) = \sum_{k=1}^{\infty} B_k \sin(kx)$$

Well ...

$$\sin^3(x) = \sin(x) \cdot \sin^2(x)$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2} \quad \text{(Derive!)}$$

$$= \frac{1}{2}\sin(x) - \frac{1}{2}\sin(x)\cos(2x)$$

?

$$\sin(x)\cos(y) = \frac{1}{2}\left(\sin(x+y) + \sin(x-y)\right)$$

$$y = 2x \qquad \text{(Derive!)}$$

$$= \frac{1}{2}\sin(x) - \frac{1}{4}\sin(3x) + \frac{1}{4}\sin(x)$$

$$= \frac{3}{4} \sin x - \frac{1}{4} \sin(3x)$$

$$B_1 = \frac{3}{4}, \quad B_2 = 0, \quad B_3 = -\frac{1}{4},$$

$$B_k = 0 \quad \forall k > 3.$$

And that's it!