Short Takes 331

Fourier series: example 1

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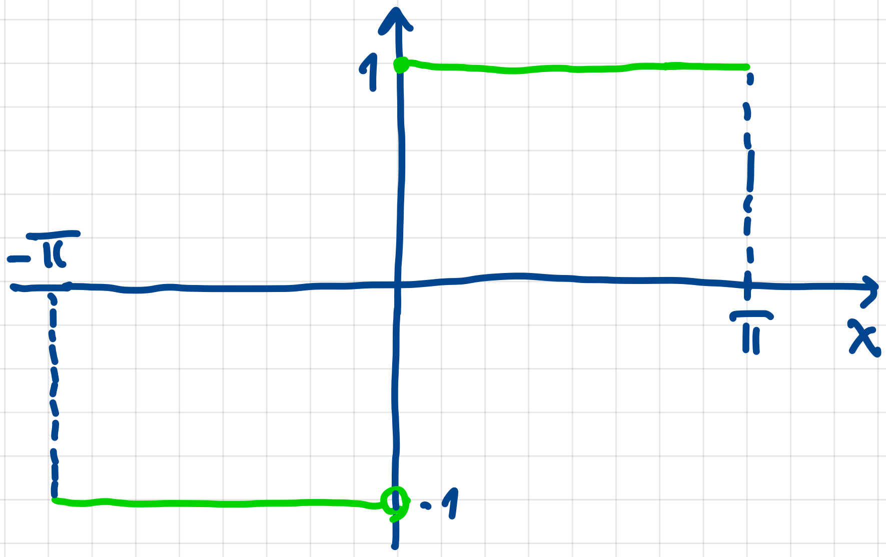
$$f(x) = \begin{cases} -1 & x \in [-\pi, 0) \\ 1 & x \in [0, \pi] \end{cases}$$



$$f(x) = \frac{A_0}{2} + \sum_{k=1}^{\infty} \left(A_k \cos\left(\frac{k\pi x}{L}\right) + B_k \sin\left(\frac{k\pi x}{L}\right) \right)$$

$$= \frac{A_0}{2} + \sum_{k=1}^{\infty} \left(A_k \cos(kx) + \underline{B_k \sin(kx)} \right)$$

where

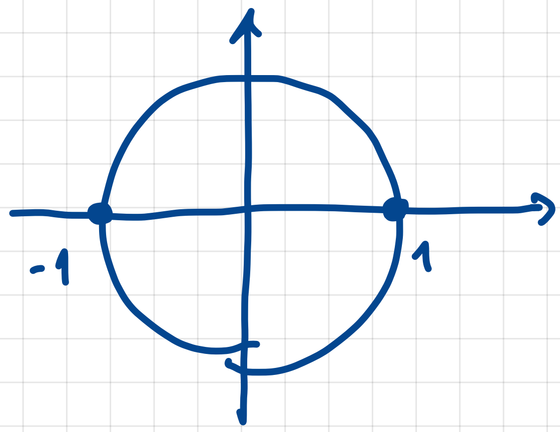
$$A_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx \longrightarrow 0$$

$$A_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(kx)\,dx \longrightarrow 0$$

$$B_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(kx)\,dx$$

$$B_k = \underbrace{\frac{1}{\pi} \int_{-\pi}^{0} (-1) \cdot \sin(kx)\,dx}_{= \frac{1}{\pi} \cdot \frac{1}{k} \cos(kx) \Big|_{-\pi}^{0}} + \underbrace{\frac{1}{\pi} \int_{0}^{\pi} (+1) \sin(kx)\,dx}_{= \frac{1}{\pi} \cdot \left(-\frac{1}{k}\right) \cos(kx) \Big|_{0}^{\pi}}$$

$$\cos(k(-\pi)) = \cos(k\pi) = (-1)^k$$



k	$\cos(k\pi)$
0	1
1	-1
2	1
$\vdots$	$\vdots$

$$\longrightarrow \quad B_k = \frac{1}{\pi}\cdot\frac{1}{k}\cdot\left(1-(-1)^k\right) - \frac{1}{\pi}\cdot\frac{1}{k}\cdot\left((-1)^k - 1\right)$$

$$= \frac{2}{\pi k}\left(1-(-1)^k\right) = \begin{cases} 0 & \text{if } k \text{ even} \\ \frac{4}{\pi k} & \text{if } k \text{ odd} \end{cases}$$

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{4}{\pi(2n+1)} \sin((2n+1)x)}$$