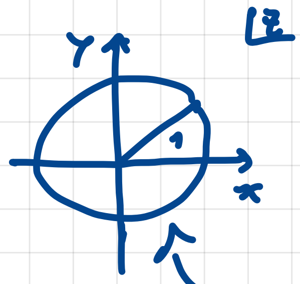
Short Takes 331

Euler’s formula

Euler's formula ... and beyond!

$$\boxed{e^{i\phi} = \cos\phi + i\sin\phi}$$



unit circle

Taylor

$$\sum_{k=0}^{\infty} \frac{(i\phi)^k}{k!} =$$

$$i^k = ?$$

$i^2 = -1$ ←

$i^3 = (-1) \cdot i = -i$ ←

$i^4 = (i^2)^2 = 1$ ←

$i^5 = i \cdot i^4 = i$ ←

$i^{2n} = (-1)^n$ even powers of i

$i^{2n+1} = (-1)^n \cdot i$ odd powers of i

$$= \sum_{n=0}^{\infty} \frac{(i\phi)^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \frac{(i\phi)^{2n+1}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{\phi^{2n}}{(2n)!} + \sum_{n=0}^{\infty} i(-1)^n \cdot \frac{\phi^{2n+1}}{(2n+1)!}$$

$$= \cos\phi + i\sin\phi$$

- What about e^z?

$$e^z = e^{x+iy} = e^x e^{iy} = e^x \cos y + i e^x \sin y \,.$$

- What about $\cosh z$ and $\sinh z$?

$$\cosh z = \frac{e^z + e^{-z}}{2} = \frac{(e^x + e^{-x})}{2}\cos y + i\frac{(e^x - e^{-x})}{2}\sin y$$

$$= \cosh x \cos y + i \sinh x \sin y$$

- $\sin z$ & $\cos z$?

$\sin z = \ldots ?$ Use $\begin{cases} e^{iz} = \cos z + i\sin z \\ e^{-iz} = \cos z - i\sin z \end{cases}$ Again from their Taylor series.

$$= \frac{e^{iz} - e^{-iz}}{2i}$$

$$= \frac{e^{i(x+iy)} - e^{-i(x+iy)}}{2i} = \frac{e^{ix}e^{-y} - e^{-ix}e^{y}}{2i} = \ldots$$

You can express using Euler's formula and

$$\begin{cases} e^{y} = \cosh y + \sinh y \\ e^{-y} = \cosh y - \sinh y \end{cases}$$

If you aim to be a physicist or a mathematician, these are the kinds of elementary operations and equations you will be expected to know very well.